(U) Measuring the Price of Anarchy via Perspective Optimization of Unmanned Vehicles in ISR Operations

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ABSTRACT

We develop a mixed integer linear program (MILP) to maximize the information gain from a team of autonomous unmanned vehicles (UxVs). Our modeling and algorithmic development enables UxVs operating in a decentralized framework to develop flight plans that simultaneously adapt to the perceived environment and support Intelligence, Surveillance and Reconnaissance (ISR) mission objectives. The mathematical formulation considers each UxV's perspective of the environment and mission, as information is only exchanged when UxVs are part of the same communication network. The main strategy is to discretize *space* and *time* to represent the potential information gain. The mathematical program is used to evaluate the "Price of Anarchy": the loss of effectiveness on the system due to the lack of overall coordination of its resources. Network connectivity is represented in the MILP by a set of binary variables. When communication links are added or removed from the problem, the structure of the connectivity matrix permits the identification of sub-networks (i.e., connected components) within the set of UxVs, allowing for an evaluation of system performance with different degrees of decentralization. Our approach is innovative in proposing a "Perspective Optimization" method as well as to measure the "Price of Anarchy" when a team of UxVs performs across multiple mission-centric ISR tasks.

Keywords: autonomous unmanned system, decentralized framework, optimization, price of anarchy

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1.0 Introduction

Military operators at all levels of Command and Control (C2) systems need to focus their attention on a wide variety of important issues to make critical decisions within operational timelines. Autonomous unmanned vehicles (UxVs) allow military personnel to spend a greater percentage of their time analyzing threat situations, as opposed to determining how to acquire information, resulting in timely decision-making that could mean the difference between mission success and failure.

We develop a mixed integer linear program (MILP) to maximize the information gain from a team of autonomous unmanned vehicles (UxVs). Our modeling development enables the unmanned vehicle system, consisting of the UxV equipped with on-board sensors and its off-board Control Station (CS), to develop flight plans that simultaneously adapt to the perceived environment and support Intelligence, Surveillance and Reconnaissance (ISR) mission objectives in a decentralized framework. The mathematical formulation considers each UxV's perspective of the environment and mission, as information is only exchanged when UxVs are part of the same communication network. Each UxV perspective of the environment forms the basis of our "Perspective Optimization" approach. The main strategy is to discretize *space* and *time* to represent the potential information gain. The mathematical program is used to evaluate the "Price of Anarchy": the loss of effectiveness on the system due to the lack of overall coordination of its resources. Network connectivity is represented in MILP by a set of binary variables, c_{ijt}. When communication links are removed (i.e., $c_{ijt} = 0$) from the original problem (i.e., centralized framework) in which all communication links are present (i.e., $c_{ijt} = 1$), the structure of the connectivity matrix allows the identification of sub-networks (i.e., connected components) within the set of UxVs, allowing for an evaluation of system performance with different degrees of decentralization. Our approach is innovative in proposing a "Perspective Optimization" method as well as to measure the "Price of Anarchy" when a team of UxVs perform across multiple mission-centric ISR tasks. The developed mathematical programming model and applied concepts can be easily extended to analyze other unmanned or manned systems.

In our model, each CS requires line-of-sight to communicate with its UxV. The range for this communication is limited. UxVs are responsible for sensing the environment. Each CS is responsible for determining the flight plans of its UxV over the planning horizon, considering mission objectives, its perspective of the environment, and the potential collected information from its UxV. Each CS can also exchange information with other "neighbor" CSs. This communication, however, is also limited. The primary objective of routing the UxVs is then to maximize the overall expected information gain considering available sensing capabilities; simultaneously, CSs' need to maximize the network connectivity with other CSs to enable information sharing over the planning horizon in the decentralized framework. Our proposed approach accomplishes these objectives, enabling unmanned vehicle systems to autonomously develop and follow flight plans that, simultaneously, adapt to the perceived environment and support mission objectives. Moreover, our approach allows us to measure the "price of anarchy" when each CS perform its own flight plan optimization.

The rest of this paper is organized as follows: Section 2.1 describes, in general, different frameworks for planning and control of autonomous systems. Section 2.2 describes recent research for planning and control assuming a decentralized framework. Section 3 describes our proposed mathematical model. In Section 4 we introduce the concept of the price of anarchy. In Section 5 the applicability of the model to measure the price of anarchy is presented. Finally, in Section 6, conclusions and future research are discussed.

2.0 Centralized, Decentralized and Hybrid Frameworks

Frameworks for planning and control of autonomous systems for many cases, similar to information fusion systems, have been classified into three types of topologies [1, 2]: (1) centralized, (2) decentralized, and (3) hierarchical (hybrid).
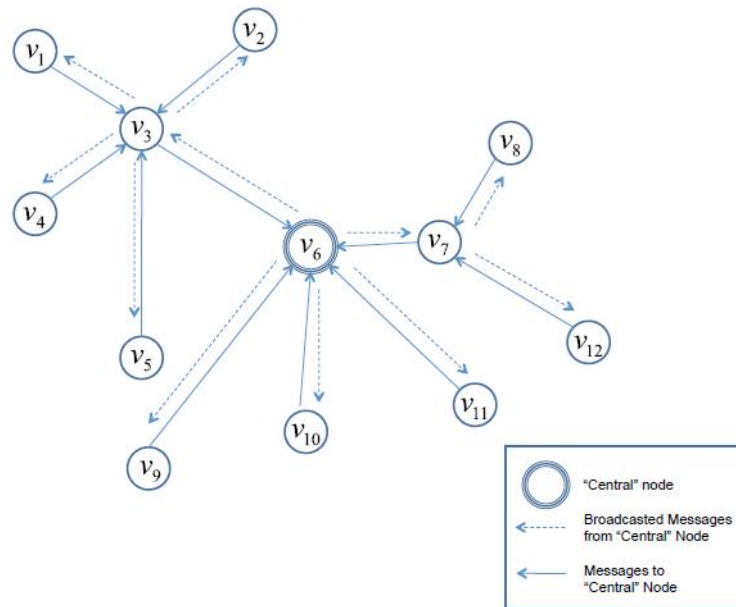

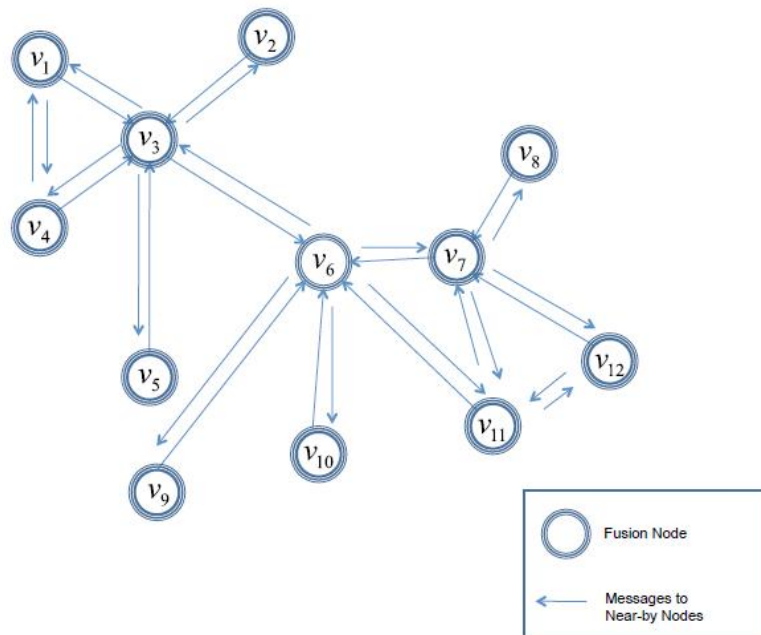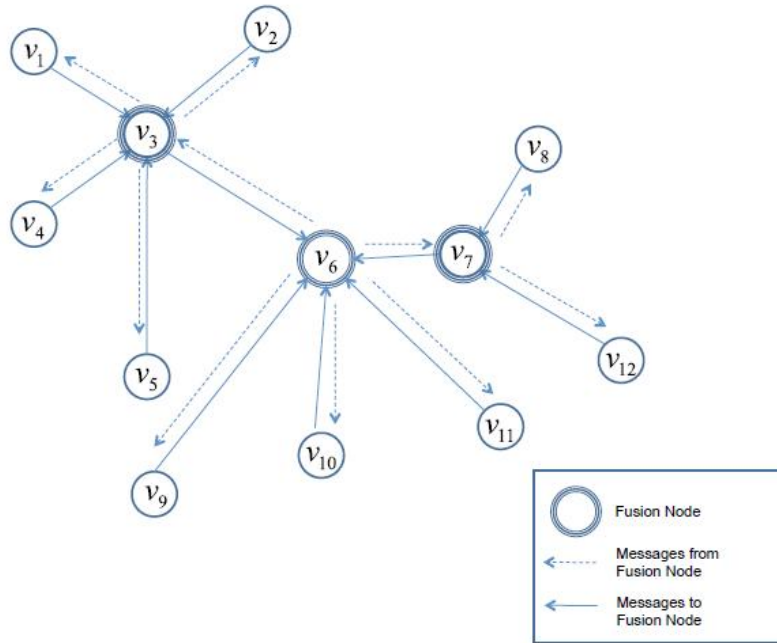**Figure 1. Example of a Centralized Framework**

Figure 2. Example of a Decentralized Framework

Figure 3. Example of a Hybrid Framework

In a centralized architecture, the information is propagated from node to node in the network until it reaches a "central" node responsible for determining and disseminating expected decisions (e.g., path definition, tasks assignments) to all lower nodes (Figure 1). This requires high computational burden at the central node and a robust and reliable communication network that allows virtually perfect information flow among all the agents in the system and the central node.

Decentralized frameworks (Figure 2) rely on local (i.e., by each agent) processing of information, including information from nearby agents, and local decision-making. This framework reduces the computational and communication requirements of a centralized framework, allowing scalability of the system to large group sizes [3]. Jameson, in [4], compared a few distributed architectures based on a set of general requirements for distributed information fusion. In his work, nodes on the network consisted of fusion centers (e.g., command and control centers) and/or sensors. For the purpose of this discussion, it is at these fusion nodes where planning and control decisions are made. The first architecture analyzed was the single composite picture implemented by the US Navy in the Cooperative Engagement Capability (CEC) system. This decentralized framework consists of high speed communication links connecting peer nodes. Each node consists of high quality sensors. All nodes in this architecture fuse data using the same algorithm so, given the low latency provided by the network, all nodes maintain virtually the same "fused" picture. A "fused" picture refers to the representation of entities (e.g., targets, assets) in the environment by combining data from multiple sensors. As would be expected, the requirements, particularly the communications bandwidth, to maintain such an accurate and high-speed network are substantial. The grapevine architecture is also a decentralized, peer to peer architecture in which each node is capable of fusing the data collected by local sensors, as well as the data received from peer nodes. At each node, a Grapevine manager is responsible for the interchange of data with peer nodes to mitigate the communication bandwidth requirements placed on the CEC network. This manager evaluates the information needs and capabilities of the peer nodes and, as data is received, it is propagated to the appropriate node. This is referred to as an intelligent push of data. The Grapevine manager on each node is also responsible for communicating the local information data needs to peer nodes. Since the peer nodes will identify information to satisfy those needs, this is referred to as an intelligent pull of data. Finally, Jameson describes the Distributed Hierarchical Information Fusion architecture. The nodes on

this network correspond to military units in a command and control hierarchy. Each node is responsible for propagating the collected information to its parent and children nodes. Since data is propagated only to adjacent fusion nodes (Figure 3), flow of information is faster than in certain centralized architectures (e.g., Figure 1).

Most planning and control algorithms assume a centralized framework. When the challenges of decentralized frameworks are addressed, architectures such as the one implemented by CEC are usually assumed: each UxV collects information from on-board sensors and, over a low latency network, information is exchanged with neighbor UxVs (i.e., peer nodes). Information is processed and trajectories are updated, as appropriate.

2.1. Algorithms for Planning and Control Systems

Research in the area of planning and control systems has resulted in the development of algorithms assuming, mostly, a centralized framework [1, 2]: information is collected in a single, central node and optimal or near-optimal plans are defined and communicated back to the agents (e.g., UxVs) in the system. A smaller fraction of the research in this area has been concerned with the coordination of resources in a decentralized environment.

Jin, Minai and Polycarpou, in [5], considered two classes of UAVs, target recognition UAVs and attack UAVs, for the search-and-destroy problem over an area. All UAVs were assumed to have sensors needed for search. A distributed assignment, mediated through centralized mission status information was developed. At each potential target location (environment was discretized as a set of cells), UAVs can Search, Confirm, Attack, perform Battle Damage Assessment (BDA), or Ignore. A centralized information base kept essential information updated for the coordination of the UAVs. Information included, for each target location, the target occupancy probability, certainty, task status, and assignment status. In addition, the information base included state information for each UAV. A set of rules, based on the information contained in the information base, determined the assignment of tasks to UAVs. Each UAV accessed and updated the information base at each step. Two measures of performance were considered to evaluate the proposed algorithm: the time needed to neutralize all *a priori* known (stationary) targets, and the total number of steps needed to bring all cells to the Ignore status.

Shetty, Sudit, and Nagi, in [6], considered the routing of multiple unmanned (combat) vehicles to service multiple potential targets in space. They formulated this problem as a Mixed-Integer Linear Program (MILP), and decomposed the problem into: (1) the vehicle to target assignment problem, and (2) determining the tour for each vehicle to service their assigned targets. Each problem was solved using a tabu-search heuristic.

A modeling framework for the dynamic rerouting of a set of heterogeneous vehicles was presented by Murray and Karwan in [7]. Vehicles were constrained by fuel- and payload- capacity. The defined MILP maximizes overall mission effectiveness and minimizes changes to the original vehicle task assignments (i.e., the previous solution). Tasks were characterized by priority values, service duration, limits on the number of resources that may perform them, precedence relationships among tasks, and multiple time windows in which resources could be assigned to the tasks. In addition, tasks were classified as required or optional. Vehicles were characterized by a value indicating the resource's relative capability of performing a task.

Several authors (e.g., [8] – [10]) have taken an information-theoretic approach to the resource allocation problem. From this perspective, the purpose of the resource management algorithm is to reduce the uncertainty about the environment. In [8], McIntyre and Hintz demonstrated the effectiveness of this approach for sensor management on the problem of searching and tracking targets. For this problem, the area of operation was represented as a grid divided into m x n cells. Information about targets was represented as discrete probability density function (pdf) on the m x n area. The pdf represented the sensors' estimate of the location of the targets. Two types of uncertainty where considered on this problem: (i) location of undetected targets, and (ii) estimation of target state vectors. The manager decided which sensor to use and whether to continue tracking a target (already represented as a track) or to search for new ones. When a cell was observed by a sensor, the amount of information gained was defined as the change in entropy prior to and proceeding a sensor measurement. The information gained by observing a cell on the grid depended on the probability of getting detection or not. The information gained by updating a (detected) target state vector considered a norm of the respective track's error covariance matrix. The sensor management control algorithm consisted of comparing the potential information gain from each sensor and target combination. Once a target was detected, the amount of information gain was computed and a decision on whether to update the track or to search was made. If search was decided, the cell with the highest probability of detecting a target was determined and that is where a sensor was aimed. Also using an information-theoretic approach, Kreucher, et al. in [10], presented their results on a decentralized sensor management algorithm.

Hirsch, et al., in [11], mathematically formulated the problem of dynamically tracking targets of interest by a set of autonomous UAVs in a centralized cooperative control framework. A decentralized control approach for UAVs with the goal of tracking moving ground targets was developed by Hirsch, Ortiz-Peña and Eck in [12]. Targets and UAVs were moving through an urban domain, simulated as a set of buildings. The shape and location of each building was assumed to be known by each UAV. Areas in the urban domain in which an accurate representation of the ground targets was more important were represented by an importance function. This function was modeled as the sum of Gaussian probability density functions (each density function represented an individual area of importance). The vehicles operate in a decentralized manner, in which each UAV was responsible to plan its route to maintain an accurate representation of detected targets. A non-linear optimization problem was defined and solved at each time step of the duration of the simulation. A Continuous Greedy Randomized Adaptive Search Procedure (C-GRASP) [13, 14] was utilized to approximately solve this optimization problem.

The vehicles were modeled as non-holonomic point masses on a two-dimensional plane with a minimum turning radius (i.e., a Dubins vehicle [15]), and a minimum and maximum speed. Communication among UAVs was restricted to a maximum communication range, beyond which UAVs could not share information. A minimum distance among UAVs and between buildings was also considered as a collision avoidance mechanism. UAVs were assumed to be flying at a constant altitude, below the height of the buildings. Additional constraints in the formulation included line of sight to targets due to the presence of buildings, formulated through the use of Plüker coordinates [16, 17] and detection range limitations. Each UAV operated its own dynamic feedback loop in which, at each time step, it moved according to its current flight plan, sensed the environment with on-board sensors, received and shared collected information of the environment with neighbor UAVs (if any) and, when required, planned its next flight path for a fixed number of time steps.

In [18], Hirsch, Ortiz-Peña and Sudit studied the effects of this decentralized control approach for the cooperative tracking of ground targets in an urban environment, as a function of the number of UAVs. It was shown, experimentally, that the decentralized approach exploits the availability of multiple UAVs by defining routes that resulted in an accurate representation of the targets.

In [19], Hirsch and Ortiz-Peña extended their work in [12] by considering the decentralized control of a set of autonomous UAVs consisting of two sets of UAVs (determined *a priori*):

1. a set of *low-level* UAVs responsible for sensing the environment with the goal of accurately tracking targets of interest in the urban domain, and

2. a set of *high-level* UAVs responsible for providing the communication back-bone for the autonomous low-level UAVs tracking the targets of interest.

Low-level UAVs were modeled as in [12], with the additional constraint that they could only communicate with *high-level* UAVs. *High-level* UAVs, also in a decentralized cooperative control framework, maximized the potential communication of the *low-level* UAVs by planning routes that would keep them within communication range to both, *low-level* UAVs, and other *high-level* UAVs. A non-linear optimization problem was defined and solved (using the C-GRASP heuristic) at each time step of the simulation. This optimization problem maximized the number of potential direct connections of the UAVs over the planning horizon. *High-level* UAVs were assumed to be flying at a higher altitude than the *low-level* UAVs and the buildings. The created communications back-bone by the *high-level* UAVs was the mechanism by which *low-level* UAVs shared, with other *low-level* UAVs, the information on the targets they were tracking.

3.0 Model Description and Assumptions

Effective utilization of a set of unmanned vehicle systems, with limited capabilities, is required in order to fulfill a set of mission objectives efficiently. Following the approach studied in [19], we considered a set of unmanned vehicle systems consisting of an unmanned vehicle and a control station. The UxV is assumed to follow, autonomously, the trajectories defined at its CS. It is further assumed that the UxV has to remain within communication range of its CS and cannot communicate with other CSs in the area of operation. The control station is responsible for receiving information from the sensors on-board its controlled UxV and from neighbor CSs. The communication range to share information with other CSs might be different than the required communication range to control and receive its UxV's collected information.

In our algorithmic development, we assumed the effectiveness of each sensor, for each mission, is known *a priori*. This effectiveness may vary over time. The area of interest (i.e., area of operation) is represented by a grid in which each cell represents a specified region. Time is discretized and, at each time slice, the UxV may be assigned one (and only one) cell of a subset of all cells in the grid, depending on its previous assignments.

3.1.1 Model parameters

The following parameters are defined:

$$I \equiv \text{set of UxVs, indexes } i, j = \{1, 2, \ldots, |I|\}$$

$$R \equiv \text{set of collection requirements (i.e., mission), index } r = \{1, 2, \ldots, |R|\}$$

$$K \equiv \text{set of grid cells, indexes } k, k', k'' = \{1, 2, \ldots, |K|\}$$

$$T \equiv \text{set of time slices, index } t = \{1, 2, \ldots, |T|\}$$

$$x_{ik0} \equiv \begin{cases} 1 & \text{if UxV } i \text{ is at cell } k \text{ at the time of planning} \\ 0 & \text{otherwise} \end{cases}$$

$$x_{ik,-1} \equiv \begin{cases} 1 & \text{if UxV } i \text{ is at cell } k \text{ at the time slice prior to the time of planning} \\ 0 & \text{otherwise} \end{cases}$$

w_{rt} $\equiv$ weighting parameter of collection requirement r at time slice t

$\eta_{kk'}$ $\equiv$ Euclidean distance between cells k and k', $\eta_{kk'} \geq 0$ $\quad \forall k, k'$

CR_i $\equiv$ communication range of UxV i's Control Station (CS) to UxV, $CR_i > 0$ $\quad \forall i$

$\overline{CR}_i$ $\equiv$ communication range of UxV i's CS to other CSs, $\overline{CR}_i \geq 0$ $\quad \forall i$

$\psi_{ik''k'k}$
$\equiv \begin{cases} 1 & \text{if UxV } i \text{ at cell } k'' \text{ at time slice } t\text{-2, at cell } k' \text{ at time slice } t\text{-1 can be assigned to cell } k \text{ at time slice } t \\ 0 & \text{otherwise} \end{cases}$

$\varphi_{ik''k'k}$
$\equiv \begin{cases} 1 & \text{if CS } i \text{ at cell } k'' \text{ at time slice } t\text{-2, at cell } k' \text{at time slice } t\text{-1 can be assigned to cell } k \text{ at time slice } t \\ 0 & \text{otherwise} \end{cases}$

$e_{irkk't}$
$\equiv$ effectiveness of the team of UxVs on cell k for collection requirement r at time slice t,

  when UxV i is assigned to cell k'

3.1.2 Main Decision Variables

x_{ikt} $\equiv \begin{cases} 1 & \text{if UxV } i \text{ is assigned to cell } k \text{ at time slice } t \\ 0 & \text{otherwise} \end{cases}$

y_{ikt} $\equiv \begin{cases} 1 & \text{if UxV } i\text{'s Control Station (CS) is assigned to cell } k \text{ at time slice } t \\ 0 & \text{otherwise} \end{cases}$

f_{rkt} $\equiv$ potential information gain from cell k for collection requirement r at time slice t

d_{rkt} $\equiv$ increase in information value from cell k for collection requirement r at time slice t

g_{rkt} $\equiv$ information gain from cell k for collection requirement r at time slice t

Δ_{ijt} $\equiv$ distance from UxV i's CS to UxV j's CS at time slice t

Ω_{ijrt} $\equiv$ value of information gain for collection requirement r by UxV j,

  when CS i is connected to CS j at time slice t

c_{ijt} $\equiv \begin{cases} 1 & \text{if UxV } j\text{'s CS and UxV } i\text{'s CS are within communication range at time slice } t \\ 0 & \text{otherwise} \end{cases}$

3.1.3 Objective Function and Constraints

$$\alpha \max \sum_r \sum_t w_{rt} \sum_k g_{rkt} +$$

$$(1 - \alpha) \min \max_{r,k,t} \{f_{rkt}\} +$$

$$\beta \max \sum_i \sum_{j, j \neq i} \sum_r \sum_t w_{rt} \Omega_{ijrt}$$

(1)

The objective function in equation (1) consists of three terms:

(1) $\max \sum_r \sum_t w_{rt} \sum_k g_{rkt}$, which maximizes the potential information to be gained by the set of UxVs;

(2) $\min \max_{r,k,t} \{f_{rkt}\}$, which minimizes the maximim value of potential information in the area of interest; this term is required to prevent the UxVs from loitering on cells with low potential information gain until other cells increase their value; and

(3) $\max \sum_i \sum_{j,j \neq i} \sum_r \sum_t w_{rt} \Omega_{ijrt}$, which maximizes the connectivity of CSs by considering the collected information of its UxVs.

$$\sum_k x_{ikt} = 1, \quad \forall i, \forall t$$

(2)

Constraint (2) ensures that each UxV is assigned a (single) cell at each time slice t.

$$\sum_k \sum_{k'} \sum_{k''} l_{ik''k'kt} = 1, \quad \forall i, \forall t \,;\, k, k'', k' | \psi_{ik''k'k} = 1$$

(3a)

$$l_{ik''k'kt} \leq \frac{x_{ikt} + x_{ik',t-1} + x_{ik'',t-2}}{3}, \quad \forall i, \forall t, \forall k, k', k'' | \psi_{ik''k'k} = 1$$

(3b)

Constraints (3) ensures that each UxV i is assigned to a cell that can be reached at time slice t, given its assignment $x_{ik,t-2}$ and $x_{ik,t-1}$ at times $t-2$ and $t-1$, respectively (see Section 3.1.1 for the definition of $\psi_{ik''k'k}$).

$$\sum_k \sum_{k'} \eta_{kk'} x_{ikt} y_{ik't} \leq CR_i, \quad \forall i, \forall t$$

(4)

Constraint (4) ensures that UxV i remains within communication distance of its CS.

Similar to constraints (2) - (4), the assignment of CSs to a cell k is constrained by

$$\sum_k y_{ikt} = 1, \quad \forall i, \forall t$$

(5)

where constraint (5) ensures that each CS is assigned a (single) cell each time slice t.

$$\sum_k \sum_{k'} \sum_{k''} o_{ik''k'kt} = 1, \quad \forall i, \forall t \,;\, k, k'', k' | \varphi_{ik''k'k} = 1$$

(6a)

$$o_{ik''k'kt} \leq \frac{y_{ikt} + y_{ik',t-1} + y_{ik'',t-2}}{3}, \quad \forall i, \forall t, \forall k, k', k'' | \varphi_{ik''k'k} = 1$$

(6b)

Constraints (6) ensures that each CS i is assigned to a cell that can be reached at time slice t, given its assignment $y_{ik,t-2}$ and $y_{ik,t-1}$ at times $t-2$ and $t-1$, respectively (see Section 3.1.1 for the definition of $\varphi_{ik''k'k}$).

$$f_{rkt} = \underbrace{f_{rk,t-1}}_{\substack{\text{current} \\ \text{potential} \\ \text{information} \\ \text{gain}}} + \underbrace{d_{rkt}}_{\text{temporal}} - \underbrace{g_{rkt}}_{\text{geospatial}}$$

(7)

f_{rkt} in constraint (7) is the potential information gain for collection requirement r from cell k at time slice t. It consists of three components: for each collection requirement r, $f_{rk,t-1}$ represents the potential information remaining on cell k from the previous time slice (i.e., potential information on cell k once the UxVs were assigned to a cell and collected information at time slice $t-1$); the temporal component d_{rkt} represents the increase in potential information from time slice $t-1$ to time slice t; the geospatial component g_{rkt} represents the information gained from cell k, given the assignment of UxVs at time slice t. This is represented in Figure 4. In this figure, the path of 3 UxVs need to be defined so that the information gain is maximized. Potential information gain is represented by the color of each cell (color follows the spectrum shown in the figure in which color blue indicates low information gain and color red indicates high information gain). At each time slice, the current information gain is increased by d_{rkt}. Each configuration of UxVs (i.e., assignment of UxVs to cells), at each time slice, results in an information gain g_{rkt}, decreasing the potentially available information for the next time slice. The solution to our mathematical programming model represents the optimal path each UxV should follow so overall information gain, over the planning horizon, is maximized.
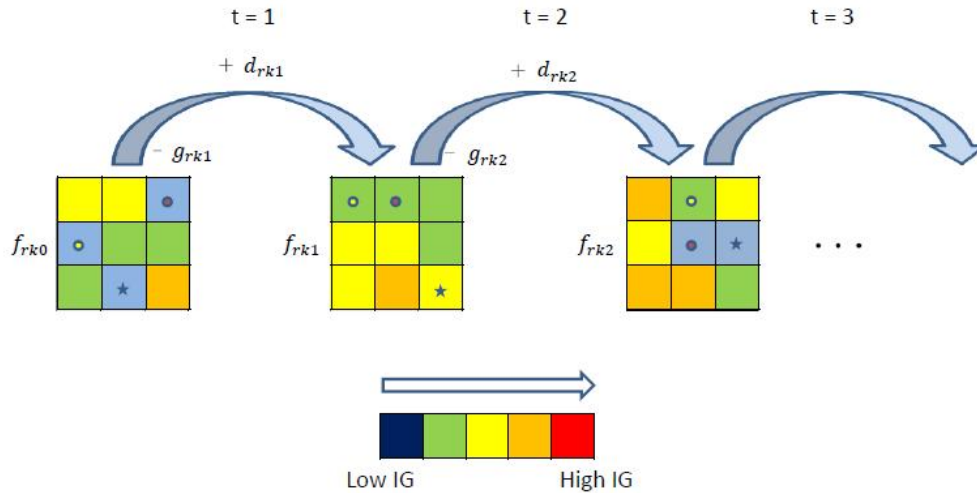

Figure 4. Relationship of Temporal and Geospatial Components on Potential Information Gain Map

g_{rkt} can be represented as

$$g_{rkt} = e_{rk}(\mathbf{x}_t) \cdot (f_{rk,t-1} + d_{rkt})$$

(8)

where $e_{rk}(\mathbf{x}_t) \in [0,1]$ represents the effectiveness of the team of UxVs gaining information on collection requirement r from cell k, as a function of the assignment $\mathbf{x}_t$.

$\mathbf{x}_t = \{x_{ikt}\}_{\forall i,t} \equiv$ vector of cell assignments at time slice t, for all UxVs. For consistency, $e_{rk}(\mathbf{x}_t)$ will be represented by e_{rkt}.

Using the Attenuated Disk Model ([20]) we can define e_{rkt} by

$$e_{rkt} = \sum_j \sum_{k'} e_{jrkk't} \cdot x_{jk't} \leq 1, \quad \forall r, k, t$$

g_{rkt} can then be rewritten as

$$g_{rkt} = e_{rkt}(f_{rk,t-1} + d_{rkt})$$

$$= \sum_j \sum_{k'} e_{jrkk't} \cdot x_{jk't}(f_{rk,t-1} + d_{rkt})$$

$$= \sum_j \sum_{k'} e_{jrkk't} \cdot x_{jk't} \cdot f_{rk,t-1} + e_{jrkk't} \cdot x_{jk't} \cdot d_{rkt}$$

$$\Rightarrow g_{rkt} = \sum_j \sum_{k'} e_{jrkk't} \cdot \theta_{irkk't} + \sum_j \sum_{k'} e_{jrkk't} \cdot \gamma_{irkk't}, \quad \forall r, \forall k, \forall t$$

where $\theta_{irkk't} = x_{ik't} f_{rk,t-1}$ and $\gamma_{irkk't} = x_{ik't} d_{rkt}$.

For communication objectives, consider

$$\Delta_{ijt} = \sum_k \sum_{k'} \eta_{kk'} y_{ikt} y_{jk't}, \quad \forall i, j, j \neq i, \forall t$$

(9)

Decision variable c_{ijt} is defined by

$$c_{ijt} = \begin{cases} 1 & \text{if } \Delta_{ijt} \leq \overline{CR_i} + \overline{CR_j} \\ 0 & \text{otherwise} \end{cases}, \quad \forall i, j, j \neq i, \forall t$$

(10)

Let

$$\Pi_{jrt} = \sum_k \sum_{k'} e_{jrkk't} \theta_{jrkk't}, \quad \forall j, \forall r, \forall t$$

(11)

where Π_{jrt} represents the contribution of UxV j to collection requirement r at time slice t. Then

11

$$\Omega_{ijrt} = \begin{cases} \Pi_{jrt} & \text{if } c_{ijt}=1 \\ 0 & \text{otherwise} \end{cases} \quad , \quad \forall i,j,j \neq i, \forall r, \forall t$$

(12)

Equations (1) – (11) were linearized. When a centralized framework is assumed, the set I includes all available UxVs to be assigned in the area of interest; when a decentralized framework is assumed, each CS will solve the programming model only considering the other CSs within communication range. While analyzing the effects of decentralization on solution quality, $\beta = 0$ and constraints (9) to (11) are not enforced: different configurations of UxVs are evaluated (i.e., $c_{ijt} = 1$ for some i, j and t) without considering the CS communication range limitations. Solutions for these configurations are then used to compute the price of anarchy (see Section 4).

4.0 Measuring the Price of Anarchy

As discussed in Section 2.2, planning and control algorithms assumed a centralized architecture. Given that reality, how can a system designer quantify the potential loss of system's efficiency when the planning and control system is implemented in a decentralized framework (compared to a centralized framework)? Having a mathematical model of the system, we are interested in measuring the effects of the lack of an overall, ``central'' controller to the optimal value that would be obtained when such centralized coordination is available. The "Price of Anarchy" (PoA) is defined as a measure on the degradation of solution quality as a centralized system moves to a more decentralized framework. The term "price of anarchy" was used in [21] to refer to the inefficiency of a system when individuals (i.e., agents) maximize decisions without coordination. Researchers have continued using this term to refer to the efficiency-loss ratio described above [22] – [27]. We will consider PoA to be defined as

$$PoA = 1 - \frac{\sum_r \sum_t w_{rt} \sum_k g_{rkt}}{\sum_r \sum_t w_{rt} \sum_k g^*_{rkt}}$$

(12)

where g^*_{rkt} refers to the information obtained by the solution of the centralized framework for collection requirement r from cell k at time slice t. As described in Section 3, w_{rt} represents a weighting parameter for collection requriement r at time slice t.

As indicated above in the proposed model, the network connectivity is represented by a set of binary variables, c_{ijt}. When communication links are removed (i.e., $c_{ijt} = 0$) from the original problem (i.e., centralized framework) in which all communication links are present (i.e., $c_{ijt} = 1$), the structure of the connectivity matrix allows the identification of sub-networks (i.e., connected components) within the set of UxVs (Figure 5). Effects on PoA will be studied as each unmanned vehicle system defines its UxV flight plan based only on the information to be obtained from its sub-network. Degrees of decentralization will be represented by redefining the structure of this connectivity matrix and the "price of anarchy" will be measured by comparing the resulting information gain against the best-case "centralized optimal solution". g_{rkt} in (12) is the resulting information gain for collection requirement r from cell k at time slice t by solving the mathematical programming model consisting of equations (1) – (9) for the particular configuration network.
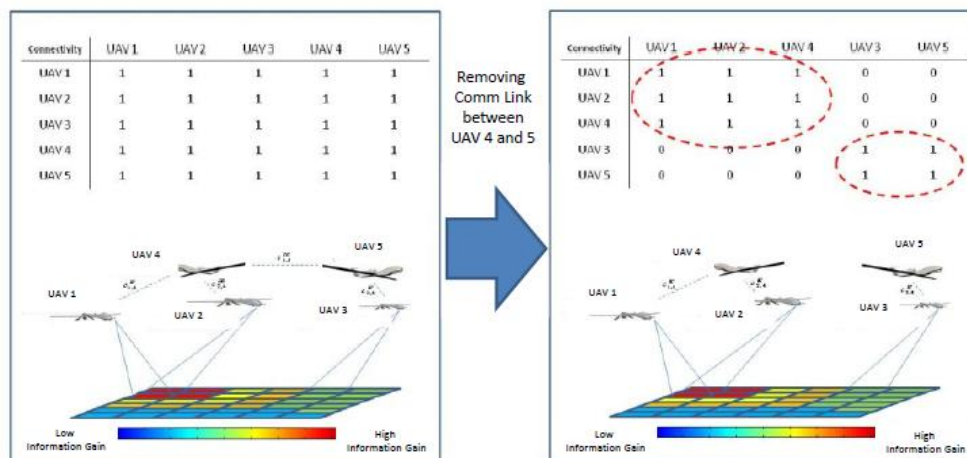
Figure 5. Example of Resulting Sub-Networks when a Communication Link is Removed

5.0 Preliminary Results

Based on the concepts described in Sections 3 and 4, a simple simulation was implemented to show the applicability and potential of our mathematical programming model to study the effects of decentralization on solution quality. We considered the assignment of 3 UxVs, particularly a set of unmanned aerial vehicles, in the area of operation represented by a grid of 5 X 5 cells shown in Figure 6. It is assumed all UAVs are autonomous, with a single on-board sensor. We considered a *search* mission in which the potential information on each cell represents the likelihood of finding a high value target on that cell. A low potential information gain region is identified on the area of operation representing, for example, a lake while looking for a car. The planning horizon was assumed to be 5 time slices.
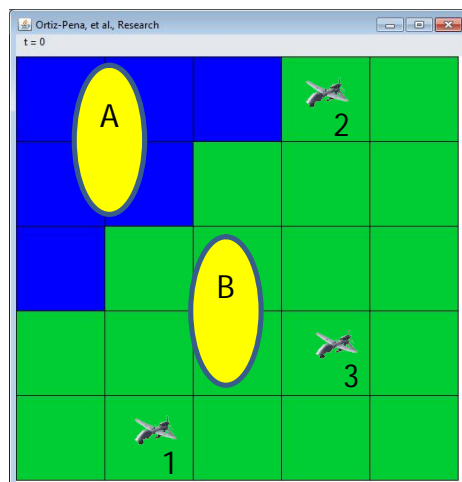

Figure 6. Area of operation:
(A) represents a low potential information gain area;
**(B) represents a high potential information gain area**

We modeled three identical UAVs:

1) On-board sensors are assumed to be radars, having a discretized effectiveness of collecting information as shown in Figure 7.

2) UAVs can only move to adjacent cells, not diagonally.

3) All unmanned aerial systems will have the same initial potential information gain map.
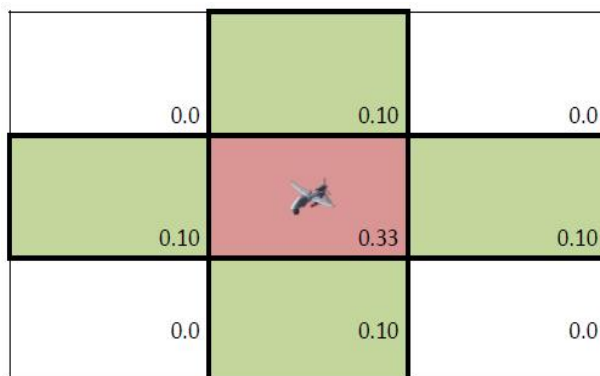
Figure 7. Sensor Model (Values represent sensor's discretized effectiveness acquiring the potential information of cell)

First, to study the price of anarchy, we considered a centralized framework in which all information collected by each UAV is assumed to be available at a "central" controller. This allows optimal coordination of all UxVs in the area of operation to maximize the information gain for the mission. For this case, in the mathematical programming model, the set *I* includes all UxVs. The routes for each UxV for this centralized framework are shown in Figures 8a – 8e. We also considered the decentralized framework in which each UxV is operating independently, with no coordination or communication among the team members. Each unmanned vehicle system is solving the mathematical programing model considering only its own UxV. The routes for each UxV for this decentralized framework are shown in Figures 9a – 9e.
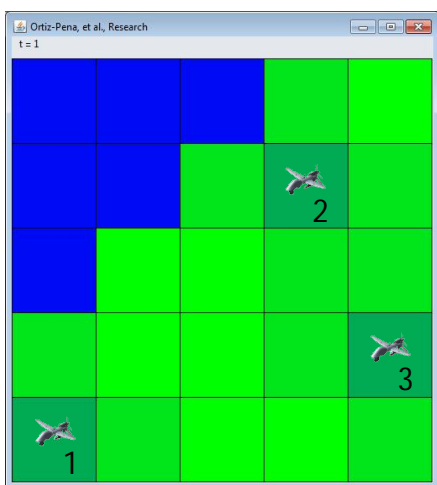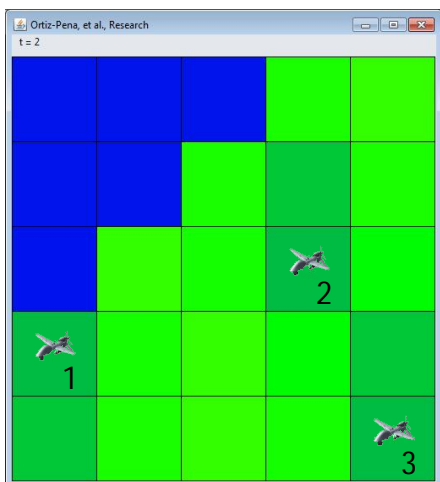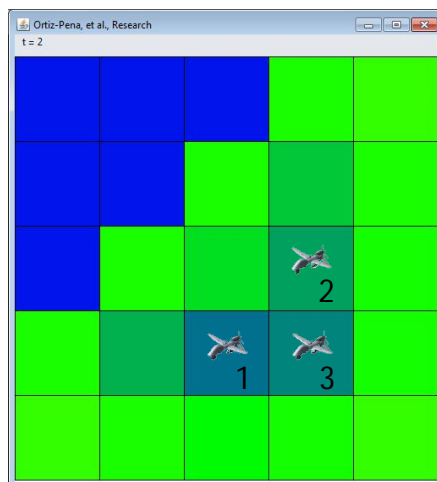

Figure 8a.
Centralized Solution (t = 1)
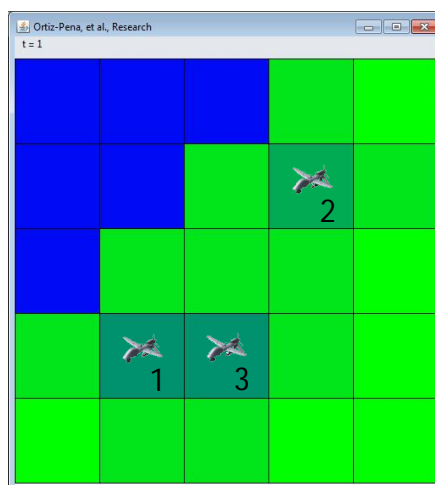
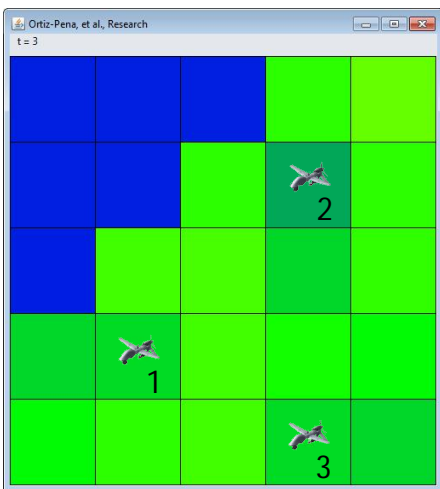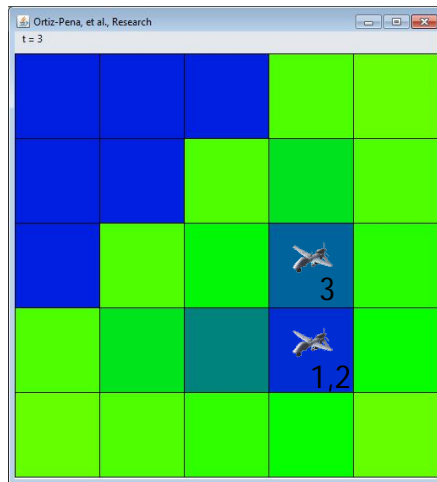**Figure 9a.**
Decentralized Solution (t = 1)

Figure 8b.
Centralized Solution (t = 2)

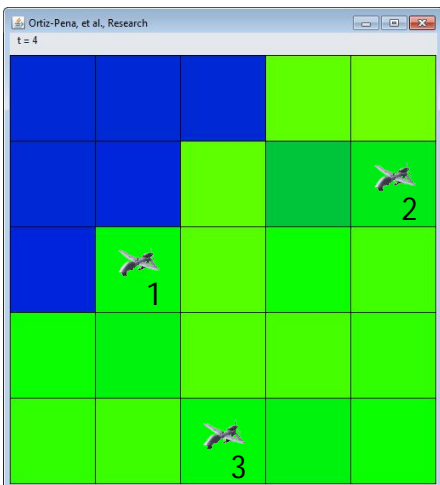**Figure 9b.**
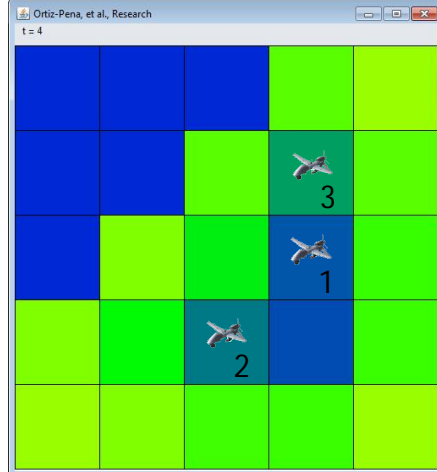**Decentralized Solution (t = 2)**
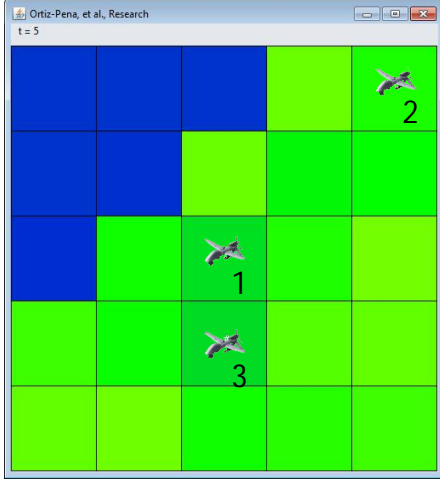

Figure 8c.
Centralized Solution (t = 3)
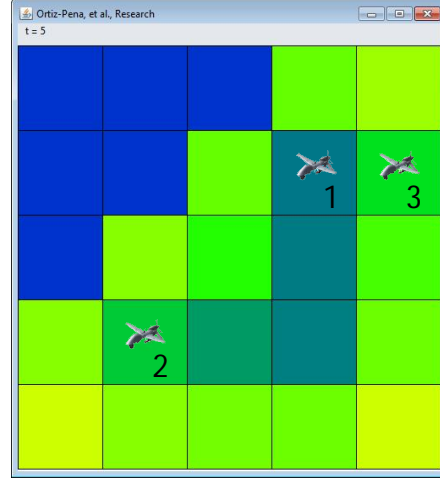

Figure 9c.
Decentralized Solution (t = 3)

Figure 8d.
Centralized Solution (t = 4)

Figure 9d.
Decentralized Solution (t = 4)

Figure 8e.
Centralized Solution (t = 5)

Figure 9e.
Decentralized Solution (t = 5)

From Figures 8a – 8e note how the centralized, coordinated solution, in general, distributes the UAVs over the area of interest. From $t=1$ to $t=4$, UAVs are assigned to areas in which their sensors' coverage do not overlap. At time slice = 5, when the sensor coverage of UAV 1 and 3 overlaps, the potential information gain in the area of operation is relatively constant and low (compared to the solution for the decentralized framework at the same time slice, Figure 9e). For the decentralized solution on Figures 9a – 9e, each UAV is trying to maximize its own potential information gain, with no consideration for the effectiveness of the other UAVs in the area of operations. In this framework, UAVs tend to travel to the same area of high potential information gain, including visiting the same cell simultaneously (see Figure 9c). Note that the solutions shown are the optimal allocation of UAVs, solving the mathematical programming model described in Section 3 using CPLEX Interactive Optimizer 12.2.

The price of anarchy for different network configurations, representing different degrees of decentralization, is shown in Table 2. UAV indexes within brackets represents connected components (e.g., [1, 2] represents that UAVs 1 and 2 communicate). For each decentralized configuration, the solution quality (i.e., information gained over the planning horizon) decreased. From Table 2, the inability to coordinate the flight path of UAV 3 (either with UAV 1, [1, 3][2], or with UAV 2, [1][2, 3]) resulted in the greatest loss of solution quality, indicating how PoA can be used to identify key communication links. Future research will include an in-depth analysis and characterization of the impact of the differences in PoA for different configurations to other measures of performance (e.g., number of detected targets, track accuracy).

Table 2. Price of Anarchy

UAV Network Configuration	PoA
[1, 2, 3] (Centralized Framework)	0%
[1, 2] [3]	6.43%
[1, 3] [2]	2.80%
[1] [2, 3]	3.80%
[1] [2] [3] (Completely Decentralized Framework)	8.31%

6.0 Conclusions and Future Needs

A mathematical model for the cooperative control of multiple autonomous UxVs involved in ISR operations was described. Using a small, simulated scenario, our model was applied to present an initial evaluation of the price of anarchy as a measure on the degradation of solution quality as a centralized system moves to a more decentralized framework. Future work will provide a more in-depth analysis of the price of anarchy and the development of heuristics that will allow us to solve the proposed mathematical model in feasible operational timelines.

The price of anarchy will be extended to consider available bandwidth, expected latencies and the value of the information flow present in the networks. A categorization of UxVs' missions should also be studied (i.e., is the price of anarchy higher on "search" missions than on "surveillance'" missions?). The dynamics of targets and the rate and requirements of new tasks on different missions might require contrasting levels of resource coordination. Having this type of analysis will be extremely beneficial to researchers developing new algorithms for autonomous unmanned vehicle systems and network infrastructures.

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